In-plane bulk-mode resonators (Lamé, breathing…)

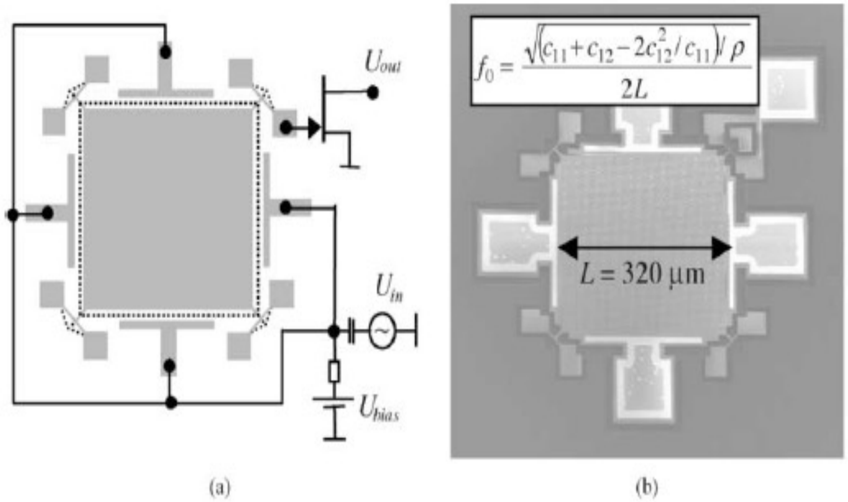
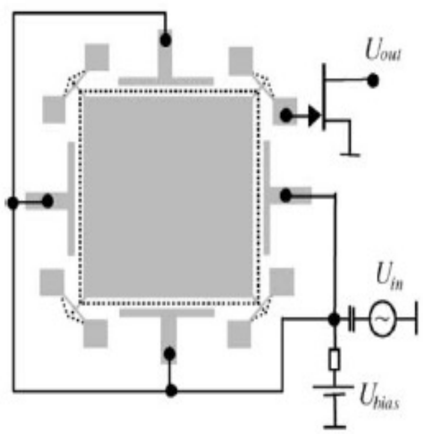


Fig. 1. Square-extensional microresonator ($f_0 = 13.1$ MHz and $Q = 130\,000$). (a) Schematic of the resonator showing the vibration mode in the expanded shape and biasing and driving setup. (b) SEM image of the resonator.

Here *h*=10 µm thick

V. Kaajakari et al., « Square-Extensional Mode Single-Crystal Silicon Micromechanical Resonator for Low-Phase-Noise Oscillator Applications », IEEE ELECTRON DEVICE LETTERS, VOL. 25, NO. 4, 2004

Bulk mode: large mass, high Q, can make as thick as technology allows



320 x 320 x 10 µm^3

For this mode, the spring constant is:

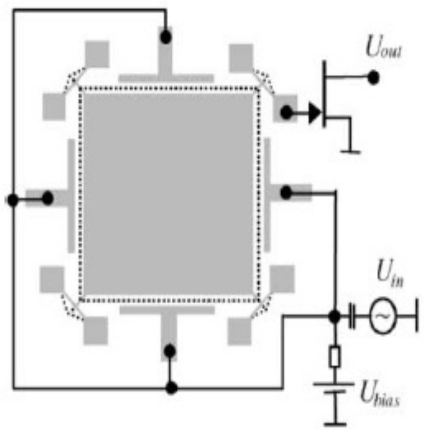
$$k = \pi^2 Y_{2D} h$$

With Y_{2D}= 181 GPa

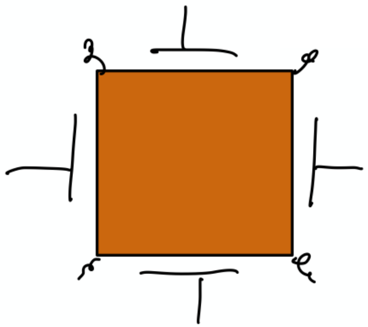
1. Compute f_{res} for given dimensions
2. How does f_{res} depend on h ?

$Y_{2D} = C_{11} + C_{12} - 2C_{12}^2/C_{11} = 181$ GPa

EPFL



320 x 320 x 10 µm^3



$m = \rho h L^2$

ρ : density
h : Si height

$k = \pi^2 Y_{2D} h \rightarrow$ prop to h

$$Y_{2D} = c_{11} + c_{12} - 2 c_{12}^2 / c_{11} = 181 \text{ GPa}$$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{\pi^2 Y_{2D} h}{\rho h L^2}} = \frac{\pi}{L}\sqrt{\frac{Y}{\rho}}$$

$\boxed{\text{no more } h \text{ dependence!}}$

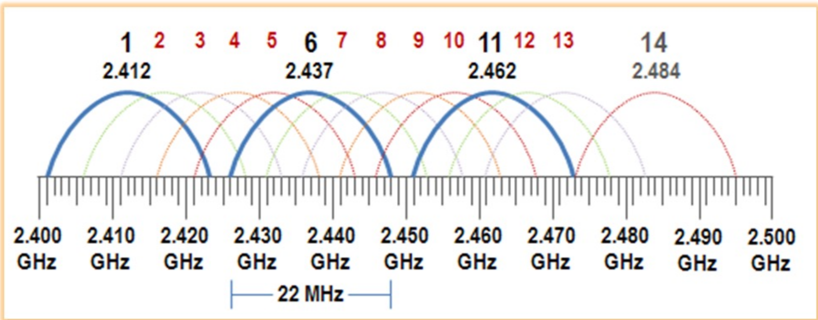
$L = 320\,\mu m$ $\quad \rho = 2300 \text{ kg/m}^3$ $\quad Y = 180 \text{ GPa}$

$$\omega = \frac{\pi}{320 \cdot 10^{-6}} \sqrt{\frac{180 \cdot 10^9}{2300}} = 8.6 \cdot 10^7$$

$$f = \frac{\omega}{2\pi} = 13.8 \text{ MHz}$$

Frequency

- If we allow max 0.1% change in frequency, how much change in L can be tolerated?
- Is this feasible? (= how well can we control L?)



If want 0.1% freq shift , what change in L ?

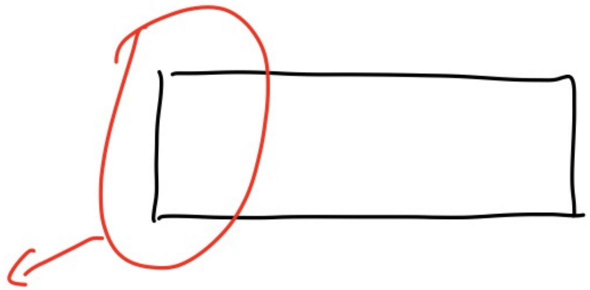
$(f_1 - f_0) = 10^{-3} f_0 \qquad f_1 = 1.001 f_0$

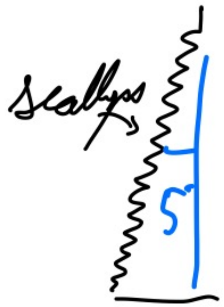
$$\frac{\pi}{L_1}\sqrt{\frac{Y}{\rho}} = 1.001\sqrt{\frac{Y}{\rho}}\frac{\pi}{L_0}$$

$L_1 = \dfrac{L_0}{1.001} \rightarrow$ 0.1% change in L

how well do we control L ?

320 ± 5 μm? depends on litho and etch





Motion due to ES force

- Compute *k*. how does this compare to spring constants you usually see in MEMS? Stiffer? Softer?

- compute the electrostatic force from the bias voltage only
  - Gap is 0.75 µm,
  - electrode length is 290 µm,
  - Bias voltage = 100 V.

- compute the electrostatic force from the AC voltage
  - Q=6200
  - V_{ac} = 1 V

- Compute the static displacement due to the bias voltage . Comments?

$$k = \pi^2 Y_{2D} h \qquad = \pi^2\, 180 \cdot 10^9 \cdot 10 \cdot 10^{-6}$$

$$= 2 \cdot 10^7 \ N/m$$ this is huge!

$$F_{ES} = \varepsilon_R \varepsilon_0 \frac{A}{d^2} V_p^2 \qquad = \frac{\varepsilon_r \varepsilon_0 \; 290 \cdot 10^{-6} \cdot 10 \cdot 10^{-6} \cdot (100)^2}{\left[0.75 \cdot 10^{-6}\right]^2}$$

STATIC

$$= \frac{1 \cdot 10^{-11} \cdot 300 \cdot 10^{-7}}{(0.75)^2 \cdot 10^{-12}} = \frac{3}{(0.75)^2} \cdot 10^{-4} \ N$$

$$= 0.5 \ mN$$

$$x = F_{ES}/k = \frac{5 \cdot 10^{-4}}{2 \cdot 10^7} = 2.5 \cdot 10^{-11} \ m$$

$$= 0.025 \ nm$$

only from V_p

$$F_{ES} \propto h$$

$$k \propto h$$

$$x = \frac{F_{ES}}{k} \propto \frac{h}{h} \propto h^0$$

no dependence of x on h
to first order

$$F_{ES} = \left[\frac{1}{2}V_p^2 + V_p V_{AC}\right]\frac{\varepsilon_0 \varepsilon_R A}{d^2}$$

$= 30\,mN$ at A_c

$$|x_{AC}| = Q\,\frac{V_p V_D}{k}\,\frac{\varepsilon_0 A}{d^2}$$

$d = 0.75\,\mu m$ $\quad A = 320\,\mu m \times 10\,\mu m$ $\quad V_{AC} = 1\,V$

$$|x_{AC}| = \frac{6200 \cdot 100 \cdot 1 \cdot 10^{-11}\; 3.2 \cdot 10^{-4} \times 10^{-5}}{2 \cdot 10^{7}\; (7.5 \cdot 10^{-7})^2} = 1.5\,nm$$

Thermal vs. mechanical energy

- At resonance, we have x_{vib} = 1.5 nm
- Compare thermal energy and mechanical energy. Your comments?
- E_{mech} : how does it scale with *h* ? With V?
- How low would the bias voltage have to be to have: $E_{mech} = 100 \times E_{thermal}$?

compare thermal E_{th} to mechanical energy E_m

$x_{vib} = 1.5\,nm$ (including Q)

$P_{drive} = 0.12\,mW$
then goes hysteretic

$k = \pi^2 Y_{2D} h$

$E_m = \frac{1}{2} k x^2 = \frac{1}{2} \pi^2 Y h x^2 = \frac{1}{2} \pi^2\, 180 \cdot 10^9\, 10 \cdot 10^{-6} \cdot (1.5 \times 10^{-9})^2$

$= 2 \cdot 10^{-11}\,J$

$E_{th} = \frac{1}{2} k_B T = \frac{1}{2} \cdot 10^{-23} \cdot 300 = 10^{-21}\,J$

$E_m / E_{th} = 2 \cdot 10^{-11} / 10^{-21} = 2 \cdot 10^{10}$

$E_{mech} \gg E_{th}$

now we set $E_m / E_{th} = 100$

$E_m \propto x^2$ and $x \propto V^2 \rightarrow E_m \propto V^4$,

so if reduce $\frac{E_m}{E_{th}}$ from 10^{10} to 10^2 $\quad V = 100 \cdot ((10^{-12})^{1/4} = 0.1V$

$$V = V_{old} \cdot \left[10^2 / 10^{14}\right]^{1/4} = 100 \cdot (10^{-8})^{1/4} = 1\ V$$

$$\left.\begin{array}{l} F_{ES} \propto h \\ k \propto h \end{array}\right) \longrightarrow x_{vib}$$ does not depend on h. so $E_{mech} \propto h$

need to choose a thick enough plate to ensure can excite the desired mode